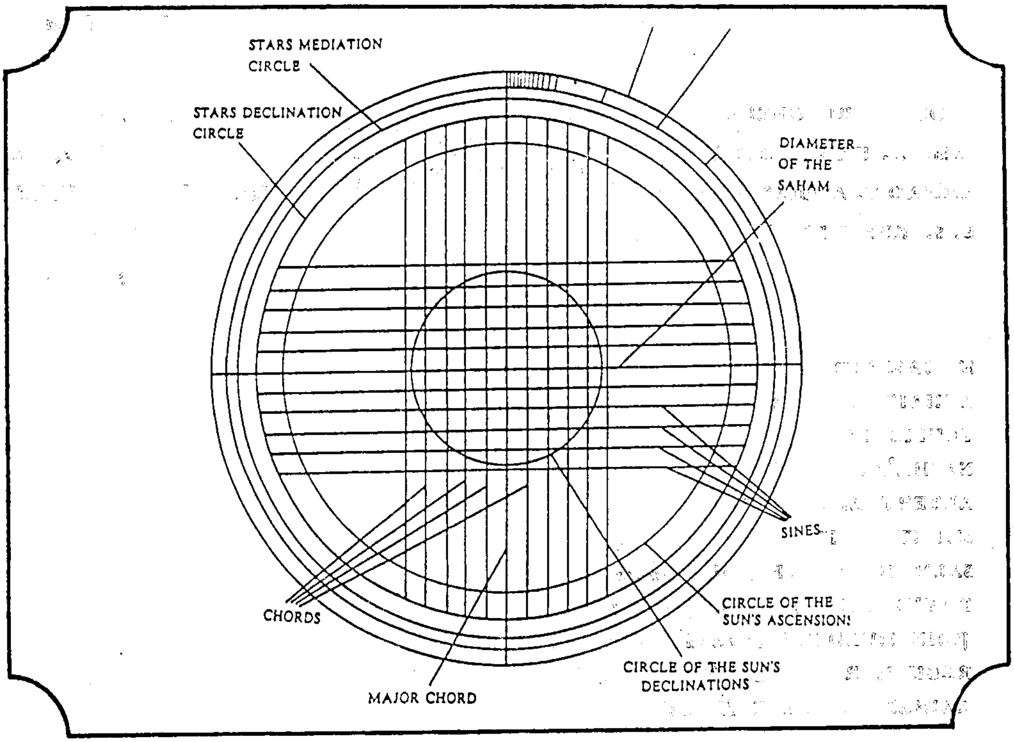
JOURNAL for the HISTORY of ARABIC SCIENCE



مجلة تاريخ العلوم العربية

ol. 12
Nos.
& 2
2001

University of
Institute for
Aleppo, Syri

TRANSFORMATION OF COORDINATES IN IBN BĀṢO'S *AL-RISĀLA FĪ'L-ṢAFĪḤA AL-MUJAYYABA DHĀT AL - AWTĀR*[*]

Emilia Calvo[**]

1 . Introduction :

Manuscript 5550 in the National Library (Bibliothèque National) in Tunis contains (in pages 50 r - 81 v) a treatise on the use of what is called *al - ṣafīḥa mujayyaba dhāt al - awtār* (plate of sines provided with chords) . The author of the treatise is mentioned on the first page . His complete name is Abū ᶜAli Ḥusayn ibn Abī Jaᶜfar (Aḥmad) ibn Yūsuf ibn Bāṣo al- Aslamī and he is described as *imām al-mu'adhdhinīn*(chief of the muezzins)*qudwat al - muᶜaddilīn* (an example for astronomers), and *amīn awqāt al - salawāt* (in charge of the times of prayer) in Granada[1] .

On page 81 the name of the copyist is also given Muḥammad b. Muḥammad , called Abū - l Ṣagīr , b. al-Ḥāy b. ᶜA bd Allah Muḥammad b.al - Ḥāy b.al - Faḏl Qāsim , and the date of copy : 1305 H.which corresponds to 1887 A.D. Therefore it is a rather late copy but this is , to the best of my knowledge , the only one preserved today .

There is no date of composition but this Ibn Bāṣo was also the author of a treatise on the use of a device that he called *al - ṣafīḥa al - jāmiᶜa li - jamīᶜ al - ᶜurūḍ* (universal plate for all latitudes) in 160 chapters .This treatise , completed in 673 H./ 1274 A.D., is preserved in three extant manuscripts now in El Escorial (MS ar . 961) , The National Library of Tunis (MS 9215) and the Royal Library of Rabat (MS 4288). The author of this second treatise is

---

* - Paper ginen at The Sixth International Symposium for The History of Arabic Science. RAS AL - KHAIMAH , 16 - 19 December , 1996 .

**- University of Barcelona .

1 - On this autour cf.SARTON, vol , III P. 696;SUTER,n°381 b,P.157; BROCKELMANN. S I, p. 869 ;SANCHEZ PEREZ, p.79;RENAUD 1932,n°381 b,p.172;RENAUD 1937,pp.1-12;MILLAS VALLICROSA 1933,pp.XXXVII-XXXVIII; MILLAS VALLICROSA 1943 - 1950 , pp. 448 - 449 ; SAMSO 1973 , pp. 176 - 182.

refered to in the first page and his complete name coincides in both cases .This second treatise was edited translated and studied several years ago . It shows that the author was aware of the *mïqät* procedures already in use among the muwaqqits of his time specially in Egypt and Syria [2].

Ibn Bäṣo's universal plate was widely diffused as we learn from the abridgements of his treatise still preserved [3]and , on the other hand , from the knowledge of this kind of projection among several astrolabe - makers who included it in their works . This , at least , can be deduced from the number of the extant plates , mainly in al - Andalus and North Africa , but also in the Muslim East .

This Ibn Bäṣo seems to be one of the two astronomers to whom Ibn al - Khaṭīb devotes some lines in his biographical work entitled *Al - Iḥäṭa fi akhbär Garnäṭa*[4] . In those lines Ibn al - Khaṭīb describes our author as *shaykh al - jamäᶜa* and says that h[illegible] author of inventions and treatises but these two treatises are the only preser[illegible] as far as I know .

2 - Description of the instrument :

The treatise on the use of the plate of sines contains 59 chapters . The first chapter is devoted to the description of the lines traced on the plate .This description is rather vague in some passages . In most cases the author limits himself to giving the name of the elements , without any indication of their appearance . Unfortunately there are no drawings on the text to clarify the nature of some of the lines engraved on the instrument . For instance , Ibn Bāṣo states that the back of the instrument is similar to the back of the astrolabe, but gives no further explanation .

Ibn Bäṣo mentions the following elements on the face of the instrument (fig . 1) [5] .

- first,the outer circle(or external circumference *dä'irat al-muḥīt),* which is divided into 360 degrees .According to the author , this circumference can correspond to the circle of the equator , the horizon circle , the mrei-

2 - Cf . Calvo , 1993 .
3 - Cf . CALVO 1991 .
4 - Cf . IBN AL - KHATiB, vol . I , p.204.
5 - Cf . fol . 51 v - 52 v

dian circle,the altitude circle and the ecliptic,depending on the operation to be performed .

- second , an inner circle , which is divided into four quadrants . In each quadrant that Ibn Bāṣo describes , every one of its ninety degrees are marked on the circle. The first quadrant is from the beginning of Capricorn to the end of Pisces.The second part goes from the beginning of Aries to the end of Gemini , and is also divided in 90 degrees . The third is not described in the text . It should go from the beginning of Cancer to the end of Virgo , and the fourth goes from the beginning of Libra to the end of Sagittarius , and is also divided in 90 degrees .

  Ibn Bāṣo states that the degree markings can be used for the ecliptic and the outer circle as well .
- then , the stars' mediation circle inside the ecliptic and , the stars' declination inside the mediation circle .

The text states that the stars are depicted between the mediation and the declination circles , and their names are written there as well . The names of the zodiacal signs are also found between these circles and the ecliptic .

the following element in the plate is the diameter of the *sahām* which is taken from the beginning of Capricorn to the beginning of Cancer .

the chords are the straight lines which cut the diameter of the *sahām* Among them is the major chord which is the diameter taken from the beginning of Aries to the beginning of Libra .

the sines are the straight lines which cut the chords .

The graduation of the *sahām* is written on its diameter . It starts at the beginning of Capricorn and finishes at the beginning of Cancer .

The graduation of the sines is written on the major chord . It starts at the centre and goes to the beginning of Aries from 0 to 90 and to the beginning of Libra also from 0 to 90 .

the text also mentions a *murï* (index) moving along a *khayṭ* (thread), which describes circles on the plate .

Finally the circle of the sun's declinations and the circle of the sun's ascensions are mentioned.The author says that the former is smaller, but gives no

other explanation of its situation on the plate . However , from the analysis of the instructions on the use of the plate which are given in the treatise for transforming coordinates , it is possible to determine the way in which these two circles are drawn . This analysis is one of the main objectives of this paper . From this description it is possible to reconstruct the aspect of the face of the instrument as seen in fig . 2.

3 - Contents of the treatise :

The use of this plate is described in chapters 2 to 59 . Ibn Bäṣo describes the following topics:

2 - 3 : Determination of sines cosines and chords.

4 - 12 : Determination of the solar degree , the latitude , the meridian altitude and the declination .

13 - 16 : Determination of shadows .

17 : Determination of the diurnal times of prayer: *al - ẓawäl* , *al - ẓuhr* and *al-^{c}aṣr.*

18 - 20 : Determination of the diurnal and nocturnal arcs and of the seasonal and equal hours .

21 - 28 : Determinations related to the time elapsed since sunrise .

29 - 32 : Calculations related to stars .

33 : Determination of morning and evening twilight .

34 - 35 : Determination of the altitude of a star from the hour and vice versa .

36 - 42 : Determination of the azimuths .

43 : Determination of longitudes .

44 - 53 : Determination related to stars .

54 - 58 : Determination of right and oblique ascensions .

59 : Determination of whether a planet is direct or retrograde .

As we can see , the first chapters are devoted to trigonometric calculations by means of plane and sphetical trigonometry . The next chapters are devoted to questions related to religious pourposes which is probably his principal interest . As I have said he is described in the introduction of the treatise as *imäm al - mu'adhdhinïn* and *amïn awqät al - ṣalwät* in Granada . He describes

how to calculate the times of the diurnal prayers (and also the nocturnal ones by determining the time of morning and evening twilight) .

4 - Transformation of coordinates in Ibn Bäso's plate of sines :

Ibn Bäṣo deals with this point in chapters 5 to 12 in which he describes how to obtain the degree of the equator from the degree of the ecliptic , that is to say , the right ascension from the longitude (in chapter 5) , and also the reverse procedure (in chapter 6) . He explains how to calculate the degree of the ecliptic from the solar declination and the reverse process (in chapters 7 to 9) ; how to obtain the latitude of a place from the meridian altitude and the declination of the sun on that day and the reverse process: the meridian altitude from the declination and the latitude of the place (in chapters 10 to 12) .

Ibn Bäso gives geometrical explanations but there are underlying trigonometrical procedures as we can see in the following examples :

In chapter 5 the right ascension is calculated from the degree of the , ecliptic . Ibn Bäṣo's instructions are (fig.3)to measure the solar longitude , λ , from point *G* which is the beginning of Aries . He obtains point *D* . The thread of the instrument will intersect the circle of ascensions , A'B'G'K, at a point *F* which determines a chord of the plate , *EH* which in turn , will intersect the sine determined by the extreme of the thread *DI*. This intersection , point H , gives the position of the thread *OH* which will determine point M on the outer circle , Arc *GM* , which equals angle *GOM* , will be the value of the right ascension , α , corresponding to the solar longitude λ given [6] .

The demonstration is as follows :

In triangle *OEF* $\sin\lambda = OE / OF$

and , since $OF = R \cos\varepsilon$, then

$$OE = R \sin\lambda \cos\varepsilon$$

In triangle *ODI*

$$\cos\lambda = OI / R \quad \text{then} \quad OI = R\cos\lambda$$

In triangle *OIH*

$Tan\ \alpha = IH / OI$ and , since $IH = OE$, by substitution

$$Tan\ \alpha = Tan\ \lambda \cos\ \varepsilon \ (= R \sin\lambda \cos\varepsilon \,/\, R\cos\lambda\ )$$

6 - Cf . fols , 53 v - 55 r

which is the underlying formula .

Chapter 6 describes the reverse procedure , that is to say , determining the degree of the ecliptic λ, from the degree of the equator (fig . 4) . In this case Ibn Bäso makes the thread coincide with a degree of the equator *GM* and determines the sine of the plate , line *IH* , which crosses the intersection of the thread and the circle of ascensions *L* . This sine line also intersects the chord corresponding to point *M*, namely *ME* , at point *N* . Then , the thread is placed on point *N* determining point *P* on the external circle which provides the value of the corresponding degree of the ecliptic [7] .

Once again the underlying formula is $tan\ \alpha = tan\ \lambda cos\ \varepsilon$ if we consider the ascension circle to have a radius $OK = Rcos\ \varepsilon$.

In chapters 7 and 8 the author determines the declination from the longitude (fig .5) . The value λ determines point *L* . The thread is placed over this point and intersects the circle of declinations at point *M*. This point determines a chord *ND* , the intersection of which with the outer circle *D* gives the value of the solar declination *GD*.

Another procedure for obtaining the chord *DN* is to determine first chord *LS*, which passes through point *L*. Then , the index of the thread is placed over the intersection of the chord with the diameter of the plate , *S*. And then , the thread is placed according to the value of the obliquity of the ecliptic *GE* . The index intersects the chord *DN*, thus giving the value of the solar declination [8].

In this case the underlying formula is

$$\text{Sin } \lambda = \text{Sin } \delta / \text{Sin } \varepsilon$$

In chapter 9 , which is the reverse of chapters 7 and 8 , Ibn Bäṣo describes how to obtain the degree of the ecliptic from the solar declination. In this case he places the alidade according to the value of the declination , $\delta = GD$. The point of intersection of the chord *DMN* with the circle of declination determ - ines a point , *M*, through which the alidade must pass to determine on the limb the arc *GL* corresponding to the value of the solar longitude [9].

7 - Cf . fols , 54 v - 55 r .

8 - Cf . fols , 55v - 56 r

9 - Cf , fol , 56 r .

Here the circle of declination is introduced . Its radius equals $r = R \sin \varepsilon$.

In fig . 5 we have that :

in triangle *ODN*

$$ON = R \sin \delta$$

in triangle *OMN*

$$\sin \lambda = R \sin \delta / r = R \sin \delta / R \sin \varepsilon$$

therefore we have that

$$\sin \lambda = \sin \delta \; / \; \sin \varepsilon$$

In chapter 10 the latitude is determined from the meridian altitude . Here the author applies to the plate the formula $h = 90 - \varphi + \delta$.

In this chapter we find a coincidence with something found in the treatise on the universal plate . The author takes into account the possibility of the sun beign situated to the north of the pole . This happens when the latitude is smaller than the declination ($\varphi < \delta$) . In this case h would be greater than 90 degrees , which is impossible . The solution is to obtain $h' = 180 - h = 90 + \varphi - \delta$. In this case he operates in the opposite quadrant of the instrument (fig . 6)[10].

In chapter 11 the meridian altitude is determined from the longitude or the declination , In chapter 12 the longitude and the declination are determined from the meridian altitude .

5 - Abridged reference :

In short , this plate makes it possible among other things , to transform coordinates and to solve problems of spherical astronomy avoiding all the trigonometric calculations involved which are implicit in the use of the instrument .

From the indications given by the author it is possible to determine how the ascension circle and the declination one are drawn. The ascension circle has a radius $r_1 = R \cos \varepsilon$ and the declinaton circle has a radius $r_2 = R \sin \varepsilon$.

A brief schema of the topics dealt with in the corresponding chapters is given below . In this analysis the trigonometrical formulae involved are ;

Chap. 5	:	$\alpha(\lambda)$	$\tan \alpha = \tan \lambda \cos \varepsilon$
Chap 6	:	$\lambda(\alpha)$	$\tan \lambda = \tan \alpha / \cos \varepsilon$
Chap 7- 8	:	$\delta(\lambda)$	$\sin \delta = \sin \lambda \sin \varepsilon$
Chap 9	:	$\lambda(\delta)$	$\sin \lambda = \sin \delta / \sin \varepsilon$

10- Cf . fol . 56r - 56v .

which is the underlying formula .

Chapter 6 describes the reverse procedure , that is to say , determining the degree of the ecliptic λ , from the degree of the equator (fig . 4) . In this case Ibn Bäso makes the thread coincide with a degree of the equator *GM* and determines the sine of the plate , line *IH* , which crosses the intersection of the thread and the circle of ascensions *L* . This sine line also intersects the chord corresponding to point *M*, namely *ME* , at point *N* . Then , the thread is placed on point *N* determining point *P* on the external circle which provides the value of the corresponding degree of the ecliptic [7] .

Once again the underlying formula is $tan\ \alpha = tan\ \lambda cos\ \varepsilon$ if we consider the ascension circle to have a radius $OK = Rcos\ \varepsilon$.

In chapters 7 and 8 the author determines the declination from the longitude (fig .5) . The value λ determines point *L* . The thread is placed over this point and intersects the circle of declinations at point *M*. This point determines a chord *ND* , the intersection of which with the outer circle *D* gives the value of the solar declination *GD*.

Another procedure for obtaining the chord *DN* is to determine first chord *LS*, which passes through point *L*. Then , the index of the thread is placed over the intersection of the chord with the diameter of the plate , *S*. And then , the thread is placed according to the value of the obliquity of the ecliptic *GE* . The index intersects the chord *DN*, thus giving the value of the solar declination [8].

In this case the underlying formula is

$$\text{Sin }\lambda = \text{Sin }\delta / \text{Sin }\varepsilon$$

In chapter 9 , which is the reverse of chapters 7 and 8 , Ibn Bāṣo describes how to obtain the degree of the ecliptic from the solar declination. In this case he places the alidade according to the value of the declination , $\delta = GD$. The point of intersection of the chord *DMN* with the circle of declination determ - ines a point , *M*, through which the alidade must pass to determine on the limb the arc *GL* corresponding to the value of the solar longitude [9].

7 - Cf . fols , 54 v - 55 r .
8 - Cf . fols , 55v - 56 r
9 - Cf , fol , 56 r .

Here the circle of declination is introduced . Its radius equals $r = R \sin \varepsilon$.

In fig . 5 we have that :

in triangle *ODN*

$$ON = R \sin \delta$$

in triangle *OMN*

$$\sin \lambda = R \sin \delta / r = R \sin \delta / R \sin \varepsilon$$

therefore we have that

$$\sin \lambda = \sin \delta \,/\, \sin \varepsilon$$

In chapter 10 the latitude is determined from the meridian altitude . Here the author applies to the plate the formula $h = 90 - \varphi + \delta$.

In this chapter we find a coincidence with something found in the treatise on the universal plate . The author takes into account the possibility of the sun beign situated to the north of the pole . This happens when the latitude is smaller than the declination ($\varphi < \delta$) . In this case h would be greater than 90 degrees , which is impossible . The solution is to obtain $h' = 180 - h = 90 + \varphi - \delta$. In this case he operates in the opposite quadrant of the instrument (fig . 6)[10].

In chapter 11 the meridian altitude is determined from the longitude or the declination , In chapter 12 the longitude and the declination are determined from the meridian altitude .

5 - Abridged reference :

In short , this plate makes it possible among other things , to transform coordinates and to solve problems of spherical astronomy avoiding all the trigonometric calculations involved which are implicit in the use of the instrument .

From the indications given by the author it is possible to determine how the ascension circle and the declination one are drawn. The ascension circle has a radius $r_1 = R \cos \varepsilon$ and the declinaton circle has a radius $r_2 = R \sin \varepsilon$.

A brief schema of the topics dealt with in the corresponding chapters is given below . In this analysis the trigonometrical formulae involved are ;

Chap. 5	:	$\alpha(\lambda)$	$\tan \alpha = \tan \lambda \cos \varepsilon$
Chap 6	:	$\lambda(\alpha)$	$\tan \lambda = \tan \alpha / \cos \varepsilon$
Chap 7- 8	:	$\delta(\lambda)$	$\sin \delta = \sin \lambda \sin \varepsilon$
Chap 9	:	$\lambda(\delta)$	$\sin \lambda = \sin \delta / \sin \varepsilon$

10- Cf . fol . 56r - 56v .

Chap 10	:	$\varphi(\delta, h)$	$\varphi = 90 + \delta - h$
Chap 11	:	$h(\lambda(\delta), \varphi)$	$h = 90 - \varphi + \delta$
Chap 12	:	$\lambda, \delta(h, \varphi)$	$\delta = h + \varphi - 90$

α : right ascension
λ : ecliptic longitude
ε : obliquity of the ecliptic (maximum declination)
φ : latitude of the place
δ : declination
h : meridian altitude

6 - Precedents of Ibn Bäs's plate of sines :

As for the precedents of Ibn Bäṣo's plate of sines , what can be said is that from the description of this plate it is evident that it is related to the sine quadrants constructed in the Islamic East from the ninth century onwards . The first known description of this instrument was by al - Khwärizmï , following his two treatises on the astrolabe [11].

In the beginning , the quadrant was an independent instrument , but from the tenth century onwards it was found on the back of the standard astrolabe .

Ibn al - Zarqälluh was the first astronomer to introduce the sine quadrant on the back of an astronomical instrument in al - Andalus , with the instrument called *al - ṣafiḥa al - zaqäliyya* . Ibn Bäṣo knew Ibn al Zarqälluh's work as is evident from the treatise on the use of his universal plate (called *al - ṣafiḥa al - jämi^ca li - jami^cal - ^curüd*) . It is possible that some elements in the treatise on the *al - ṣafiḥa al - mujayyaba* come from Ibn al Zarqälluh . But there is a difference since Ibn Bäṣo is describing a complete plate , not only a quadrant .

This plate is probably related to the instrument called the *dastür* circle , an instrument described by several astronomers in the Islamic East . Among them are Abü Sa^cïd [^cAbd al - Raḥmän b . Abï ḤaFṣ ^cOmar b . Muḥammad] al - Abharï from the thirteenth century (d . A. D. 1274)[12]. According to Schmalzl , he may have been the inventor of this instrument which is described in his work entitled *Lawämi^c al - wasä' il fï - maṭäli^c al - rasä'il* preserved in MS. 965 (acccording to Renaud)[13] of El Escorial and in Gotha 1414 . The third maqäla

11- Cf . KING , 1983 , PP. 29 - 30
12- Cf . SUTER n . 369 , SCHMALZL, PP. 62 - 63 .
13- Cf . RENAUD 1941 , p . 109

is divided into 40 chapters devoted to the dastür .

This instrument was also described by Abü - l - Ḥasan ᶜAlï al - Marräkuši , an astronomer from the thirteenth century who worked in Egypt[14], He describes it in his treatise entitled *jämiᶜal - mabädi' wa - l - gäyät fi ᶜilm al - mïqät* .

The *dastür* is also described by a muwaqqit in al -Azhar , Jamal al -dïn al - Maridini (m. 1406)[15] , in his treatise *Al - durr al - manthür fi l - ᶜamal bi - l - rubᶜal- dastür* which is preserved in several copies now in Berlin 5840 , Madrid (Esc . 931), Oxford , Paris Cairo .[16]This treatise contains 60 chapters[17].

At the end of the XVth century an instrument called *sexagenarium* was introduced in al-Andalus .[18] This instrument had two faces ,one planetary and one trigonometric ,Of arabic origin , it was used by the *muwaqqits* in Egypt and reached the Iberian peninsula in 1450 via a *faqïh* from Paterna who was in the city of Valencia at that time.This instrument was later introduced in Europe. There are two translations of the treatise describng its use , one of which was printed. This instrument was described by Poulle several years ago and presents many points of contact with Ibn Bäso's plate of sines [19].

7 - Concluding remarks

To conclude this study I would like to add two observations , one related to the instrument itself and the other to its place in the history of the trigonometrical instruments :

First , Ibn Bäṣo does not specify in the treatise the value of the obliquity of the ecliptic . But this value is not needed , since there are two circles which are traced according to it:the circle of declinations and the circle of ascensions , with which he operates when he needs to introduce this value.In the description of the instrument these two circles are introduced in a rather vague way , but a

14- Cf . SUTER n . 363.

15- Cf . SUTER n.421.

16- Sibt al - Maridïnï's grandfather . Sibt is also the author of a treatise on the use of the sine quadrant which is contained in the same Ms. 931 Escorial .

17- Cf . SEDILLOT p.88 n . 2.

18- On this instrument cf . THORNDIKE, pp. 130 - 133 .

19- On the characteristics of the sexagenarium cf . POULLE 1966 , pp. 129 - 161 ; POULLE 1980 , I pp, 417 - 444 and SAMSO 1992 , pp 216 - 217 .

study of these chapters reveals how they are traced on the plate : the circle of declinations has a radius of value *(R sin ε)* and the circle of ascensions has a radius of value *(R cos ε)* R being the radius of the plate which , as I have said , is divided into 60 parts , and ε the value of the obliquity of the ecliptic adopted by the author . But this seems to be a rather technical question and the treatise tends to avoid them in order to make it useful for non specialists .

Secondly , This instrument can be inscribed in the stream of influences East - West - East found between al - Andalus , North Africa and the Islamic East from the eleventh century onwards .

8 - Bibliography :

BROCKELMANN:C.BROCKELMANN,*GeschichtederArabischenLiteratur* - 2 vols .Weimar , 1898 ; Berlin , 1902 ; *Supplementband* .- 3 vols . Leiden , 1937 - 1942 .

CALVO , E., " La *Risälat al - ṣafiḥa al- mushtaraka ᶜalà al - shakkäziyya de Ibn al- Bannä' de Marräkush"- Al Qantara,*10(1989) pp. 21 - 50 .

.... La lámina universal de ᶜALï b. Khalaf (s.XI) en la version alfonsi y su evolucion en instrumentos posteriores" . - " *Ochava Espera* " *y* " *Astrofisica* " . Barcelona , 1990 . pp. 221 - 238 .

........" Les échos de l'oeuvre d'Ibn Bäso en Afrique du Nord". - *Actes du VII Colloque Universitaire Tuniso - Espagnol sur Le Patrimoine Andalou dans la Culture Arabe et Espagnole.* Tunis , 1991 . pp. 65 - 79 .

.... *Abü ᶜAlï al - Ḥusayn ibn Bäso (m.716 / 1316): Risälat al - ṣafiḥa al - Ŷämiᶜa li - ŷamiᶜ al - ᶜurüḍ (Tratado sobre la lámina general para todas las latitudes) . Edición crïtica , traducción y estudio por ..* Colección Fuentes Arábico - Hispanas 13 . Madrid , 1993 .

E.I.2 : Enciclopédie de l'Islam - 2ª ed , 7 vols . (to date) . Leiden , 1960 .

IBN AL - KHATïB , Lisän al-dïn, *Al-Iḥäṭa fi akhbär Garnäṭa* -Ed.ᶜAbd Alläh ᶜInän . vol I.EI Cairo , 1973.

KAḤḤALA , R., *Muᶜdjam al - Mu 'allifïn* - Damascus 1957.

AL - KATTÄNÏ , *Salwat al - anfäs* - 3 vols. Ed. lit , Fez, 1316 H.

KENNEDY , E.,S.,*The Exhaustive Treatise on Shadows by Abü al - Rayḥän Muhammad b.Ahmad al-Bïrünï*:Translation and Commentary - 2 vols. Institute

for the History of Arabic Science. Aleppo ,1976 .

........"Al - Bīrūnī on the Muslim Times of Prayer ". *The Scholar and the Saint : Studies in Commemoration of Abū' l - Rayḥān al - Bīrūnī and Khalāl al- Dīn al - Rūmī* . New York University , 1975 . pp. 83 - 94 . Repr . in *Studies on the Islamic Exact Sciences* . pp. 299 - 310 .

KING, D.A."Ibn Yūnus' Very Useful Tables for Reckoning Time by the Sun"*Archives for the History of Exact Sciences*,10 (1973)pp . 342-394. Reimpr. en *Islamic Mathematical Astronomy.* Variorum Reprints London ,1986 .

.... *Al - Khwārizmī and New Trends in Mathematical Astronomy in the Ninth Century* - New York , 1983 pp. 29 - 30 .

.... " Kibla " - *E.I.* 2ª ed .t. 5, pp. 84 - 91.

........"Universal Solutions to Problems of Spherical Astronomy from Mamluk Egypt and Syria".*A Way Prepared* . New york , London , 1988. pp.153 - 183.

.... "A Survey of Medieval Islamic Shadow Schemes for Simple Time - Reckoning ". *Oriens* , 32 (1990) pp. 191 - 249 .

.... Mīkāt " - *E. I* . 2 , t . 27 , pp. 7 - 32 .

AL - MARRAKUSHī ,Abū-l- Ḥasan ᶜAlī , *Djāmiᶜal-mabādi' wa- l - gāyāt fī ᶜilm al - mī-qāt* - Ms . 3343 Istanbul . Ed . facsímil Institut für Geschichte der Arabisch - Islamischen Wissenschaften . Frankfurt , 1984 .

MAJLUF , M., *Šadjarat al - nūr al - zakiyya* - Cairo , 1931 .

MILLAS VALLICROSA , J.M., *Don Profeit Tibbon . Tractat de l 'assafea d'Azarquiel* . Barcelona , 1933 .

.... *Estudios sobre Azarquiel* - Madrid - Granada ,1943 - 1950 .

.... *Las tablas astronómicas del rey Don Pedro el Ceremonioso* . Madrid - Barcelona , 1962 .

POULLE ,E. *Théorie des planètes et trigonometrie au XV siècle . Le Sexagénarium . Journal des Savants* , 1966 . pp. 129 - 161 .

.......*Equatoires et horlogerie planétaire du XIII au XVI siècle.* 2vols.Ginebra , 1980 .

PUIG, R . " Concerning the *safīha shakkāziyya*". *-Zeitschrift für Geschichte des*

Arabischen Islamischen Wissenschaften , 2 (1985) pp. 123 - 139 .
.... *Al - Šakkäziyya de Ibn Naqqäsh al - zarqällüh* . Edición , traducción y estudio - Barcelona , 1986 .
........"El Taqbïl ᶜalà risälat al - ṣafīḥa al - zarqäliyya de Ibn al - Bannä' de Marräkush " - *Al Qanṭara* , 8 (1987) pp. 45 - 64.
.... *Los Tratados de Construcción y Uso de la Azafea de Azarquiel* - I. H.A.C. Madrid , 1987 .
.... *Una aportación andalusí a la difusión del cuadrante de senos* - YAD - NAMA in memoria di Alessandro Bausani . Rome , 1991 . pp. 75 - 85 .

RENAUD , H.P.J.,"Notes critiques d'histoire des sciences chez les musulmans . I. - Les Ibn Bäṣo " . -*Hespéris* , 24 (1937) pp. 1 - 12.
.... "Additions et corrections à Suter". - *Isis* , 18 (1932) pp. 166 - 183 .
.... *Les Manuscrits Arabes de L'Escurial d'après Les notes de Hartwig Deren bourg , revues par...*- vol .2 ,fasc.3 , Sciences Exactes et Sciencies Ocultes . Paris 1941 .
SAMSO , J., "A propos de quelques manuscrits astronomiques des bibliothèques de Tunis : Contribution a une étude de l'astrolabe dans l'Espagne musulmane".*Actas del II Coloquio Hispano-Tunecino* , pp. 171-190 . Madrid : I.H.A.C., 1973 .
.... et al . *El legado científico andalusí* . Madrid , 1992 .
SARTON , G . *Introduction to the History of Science* - 3 vols., 5 tomos . Baltimore , 1927 - 1948.
SATMI: D.A.KING,*Studies in Astronomical Timekeeping in Medieval Islam , vol* II: *A Survey of Tables for Regulating the Times of Prayer* . preparation .
SCHMALZL . *Zur Geschichte des Quadranten bei den Arabern.* Munich , 1929 . Reprint in *A.I.O.S.*, vol . 6 (I.G.A.I.W) Frankfurt 1991 . pp. 189 - 331 .
SCHMIDT, F .*Geschichte der Geodätische Instrumente und Verfahren im Altertum und Mittelalter* . 1929 . Reprint *A.I.O.S.*, 5 (I.G.A.I.W.) Frankfurt 1991 . p. 296 Sinusquadranten , Dasturkreis , Dasturquadranten .

SEDILLOT , L. A.,Mémoire sur les instruments astronomiques des Arabes ".

Mémoires présentées par divers savants à l'Académie Royale des Inscriptions et Belles Lettres de l'Institut de France . Paris , 1844 . Reimpr . nstitut für Geschichte der Arabisch - Islamischen Wissenschaften . Frankfurt , 1984 .

SEZGIN,F., *Geschichte der Arabischen Schriftums*-Leiden V. *Mathematik* , 1974 ; VI. *Astronomie* , 1978 .

SUTER, H. " Die Mathematiker und Astronomen der Araber und ihre Werke". *Abhandlungen zur Geschichte der Mathematischen Wissenschaften* , 10. Leipzig, 1900.

THORNDIKE, L . "Sexagenarium "*Isis* , 42 (1951) pp. 130 - 133 .

WENSINK , A.J., "Mīḳāt - *E.I.* 1,t.5 pp. 492 - 493 ; *E.I.* 2,t.7 , pp. 26 - 27 .

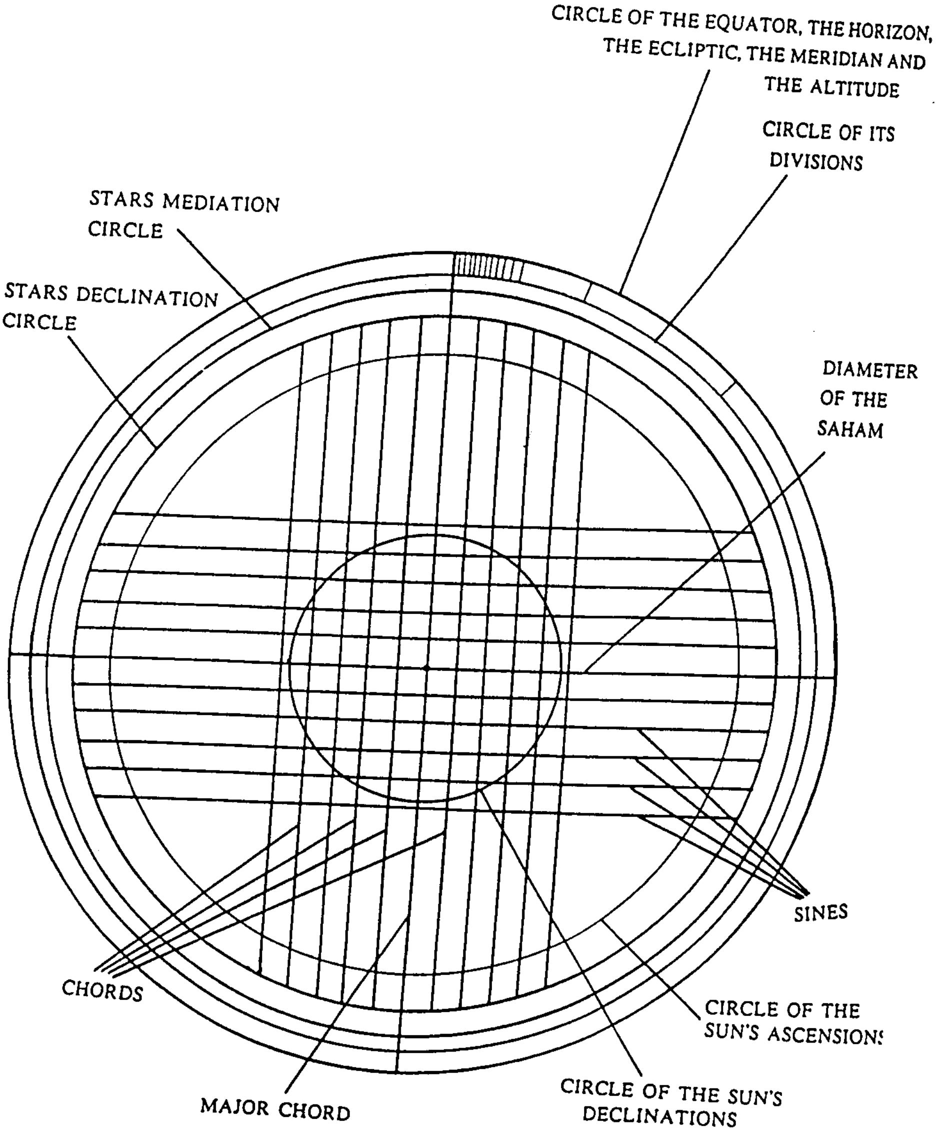


Fig. 1
Ibn Bāṣo's *al-ṣafīḥa al-mujayyaba dhāt al-awtār*
(Elements)

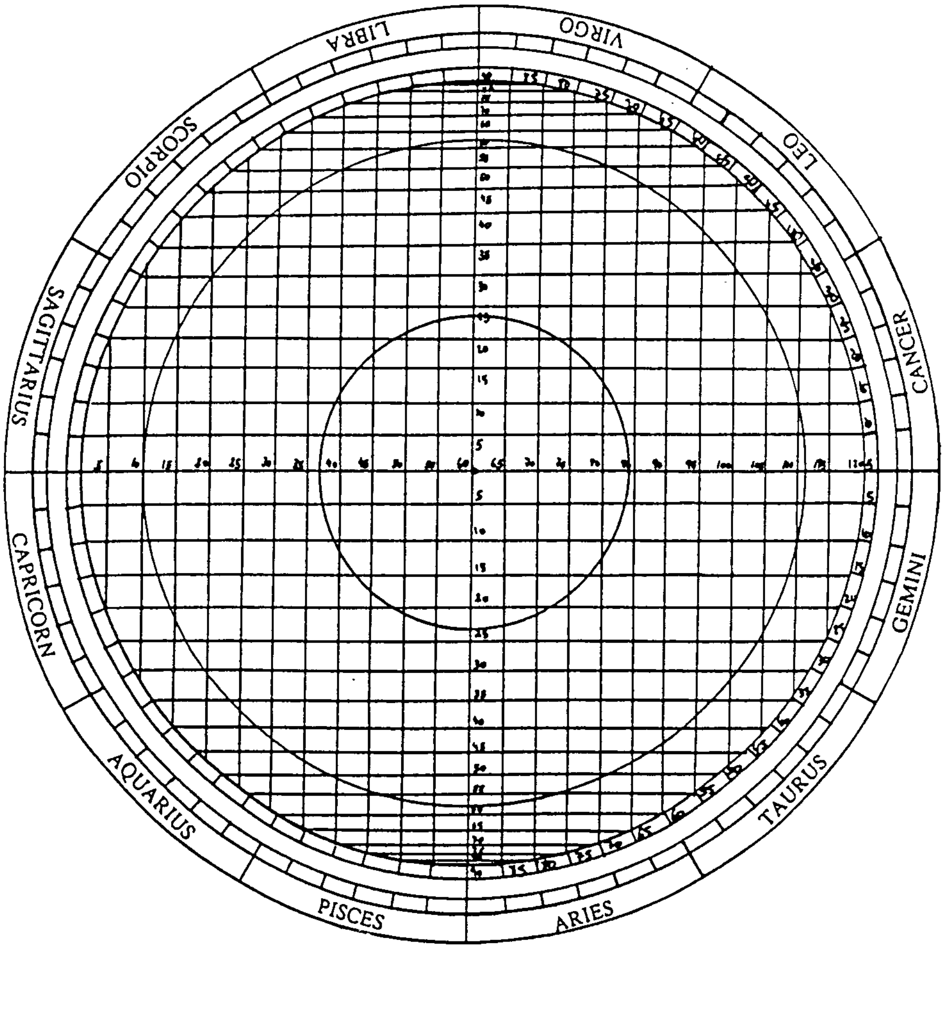
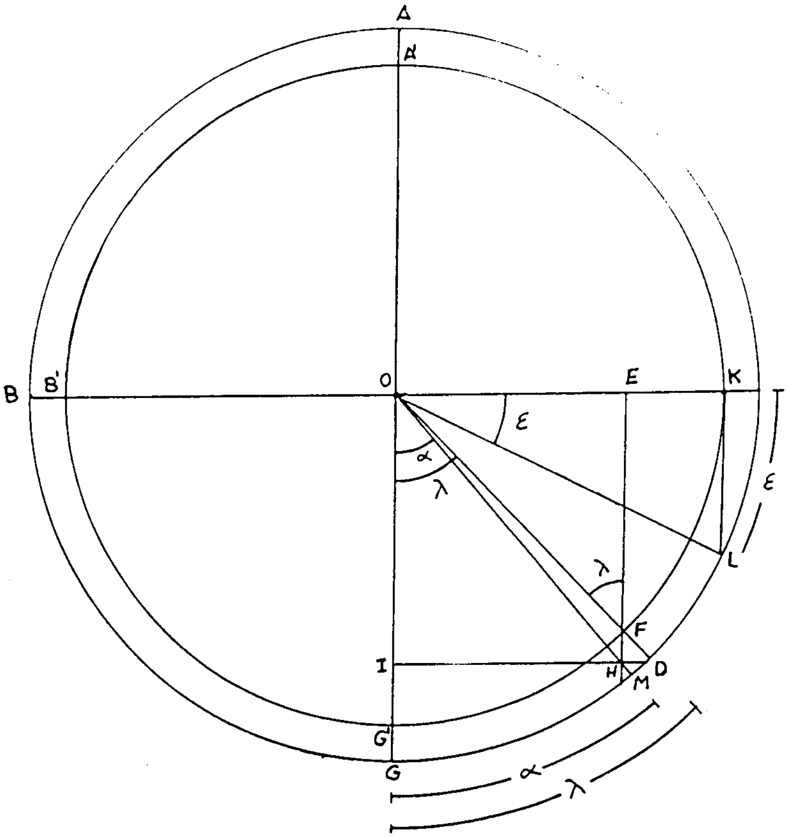
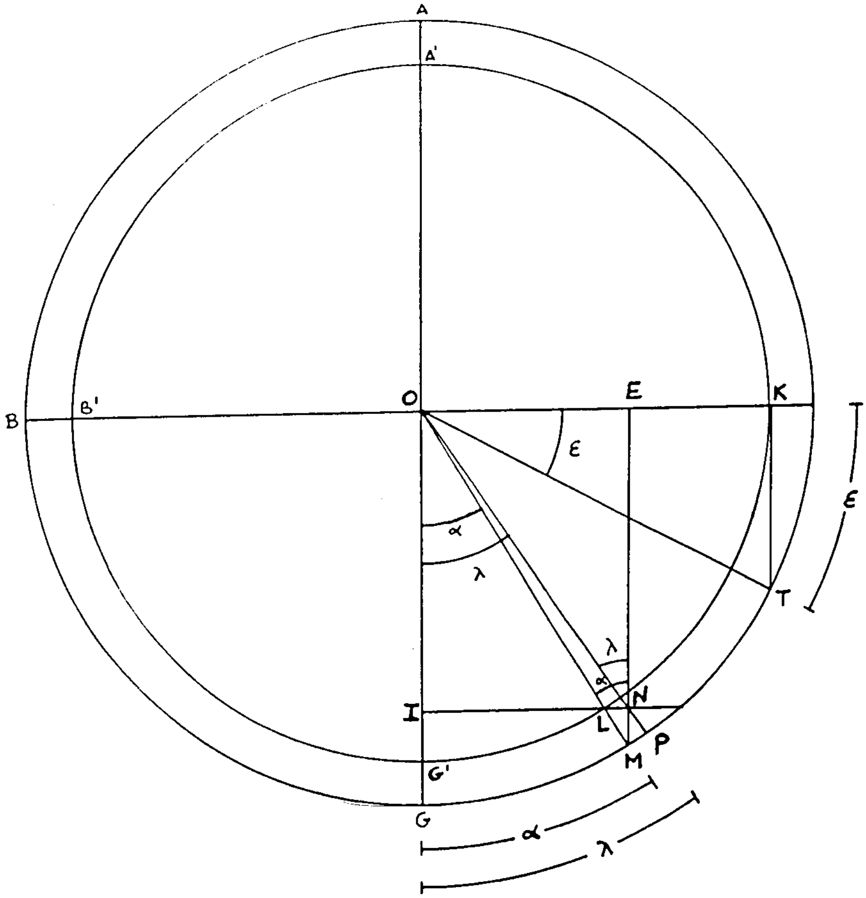


Fig. 2
Ibn Bâṣo's *al-ṣafīḥa al-mujayyaba dhât al-awtār*
(Reconstruction)



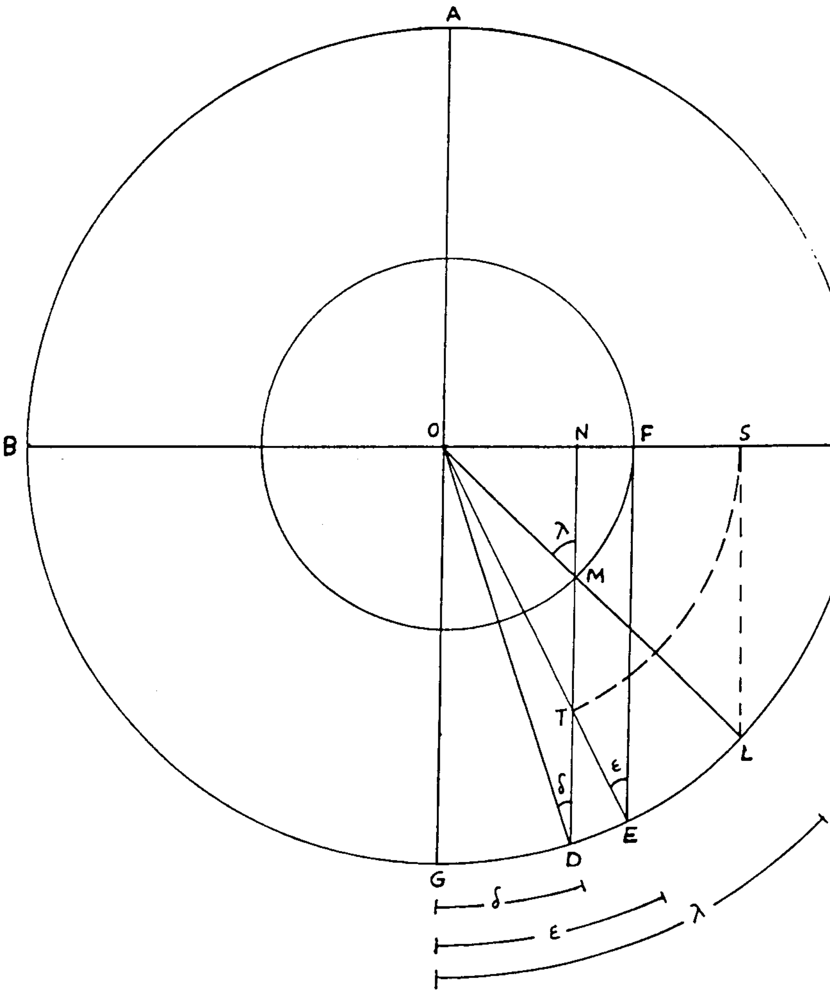
In OLK		$OK = R \cos \epsilon$		
In OEF		$\sin \lambda = OE/OF$	and	$OF = R \cos \epsilon$
	=>	$OE = R \sin \lambda \cos \epsilon$		
In ODI		$\cos \lambda = OI/R$	=>	$OI = R \cos \lambda$
In OIH		$\tan \alpha = IH/OI$	and	$IH = OE = R \sin \lambda \cos \epsilon$
	=>	$\tan \alpha = \tan \lambda \cos \epsilon$		

Fig. 3



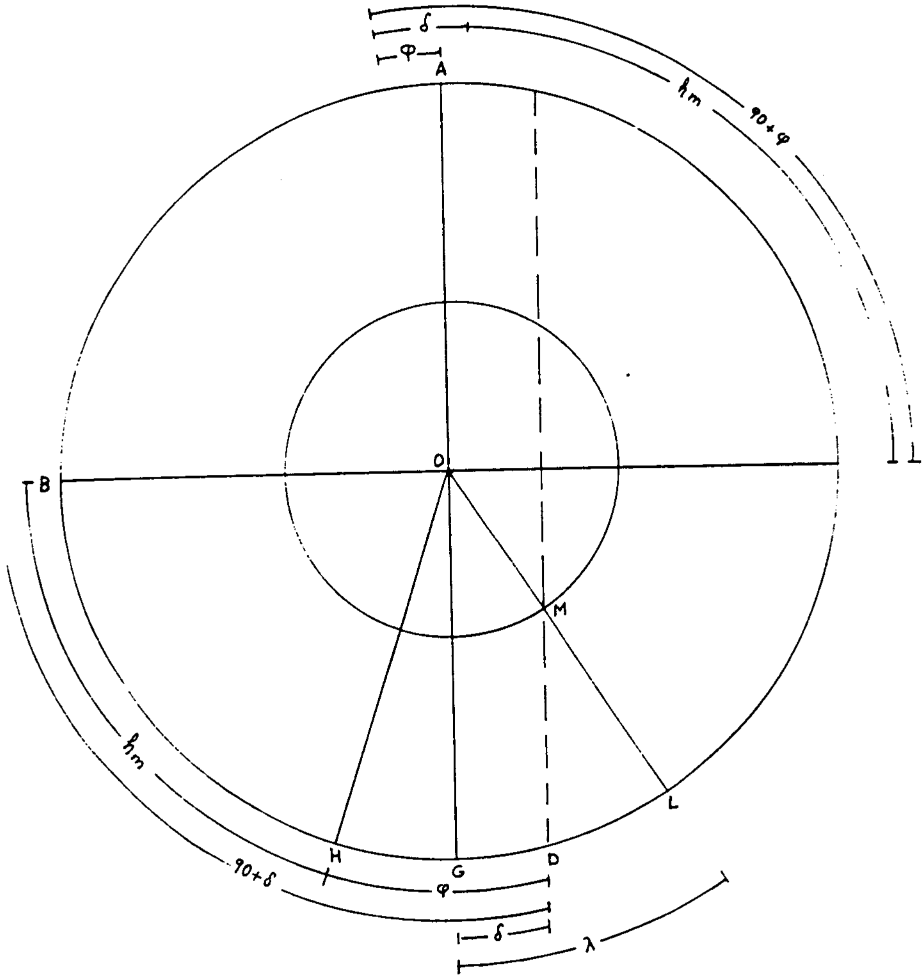
In OEM		$OE = R \sin \alpha$
In OTK		$OK = R \cos \epsilon$
In OIL		$\cos \alpha = OI/OL \qquad OL = OK = R \cos \epsilon$
	=>	$OI = R \cos \epsilon \cos \alpha$
In OIN		$\tan \lambda = OE/OI = R \sin \alpha / R \cos \epsilon \cos \alpha$
	=>	$\tan \lambda = \tan \alpha / \cos \epsilon$
	or	$\tan \alpha = \tan \lambda \cos \epsilon$

Fig. 4



In	OEF	$\sin \epsilon = OF/R$	then	$OF = R \sin \epsilon$
In	ODN	$\sin \delta = ON/R$	then	$ON = R \sin \delta$
In	OMN	$\sin \lambda = ON/OM$	and	$OM = OF = R \sin \epsilon$
	=>	$\sin \lambda = \sin \delta / \sin \epsilon$		

Fig. 5



When $\varphi < \delta$

$$h_m = 90 - \varphi + \delta.$$

$$h'_m = 180 - h_m = 90 + \varphi - \delta$$

Fig. 6